

BRST invariant effective action of shadow fields, conformal fields, and AdS/CFT

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Abstract

Totally symmetric arbitrary spin massless and massive fields in AdS space are studied. For such fields, we obtain Lagrangians which are invariant under global BRST transformations. The Lagrangians are used for computation of partition functions and effective actions. We demonstrate that BRST invariant bulk action for massless field evaluated on the solution of Dirichlet problem for gauge massless fields and Faddeev-Popov fields leads to BRST invariant effective action for canonical shadow gauge fields and shadow Faddeev-Popov fields, while the BRST invariant bulk action for massive field evaluated on the solution of Dirichlet problem for gauge massive fields and Faddeev-Popov fields leads to BRST invariant effective action for anomalous shadow gauge fields and shadow Faddeev-Popov fields. The leading logarithmic divergence of the regularized effective action for the canonical shadow field leads to simple BRST invariant action of conformal field. We demonstrate that the Nakanishi-Laudrup fields entering the BRST invariant Lagrangian of conformal field can geometrically be interpreted as boundary values of massless AdS fields.

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1 Introduction

Slavnov-Taylor identities [1] and BRST approach [2] play important role in studies of the renormalizations of gauge theories. In the quantum field theory, Heisenberg equations of motion are not well defined. Therefore, in studies of various theories of interacting quantized gauge fields, usually one concentrates on the study of S-matrix. In such studies, it is the Slavnov-Taylor identities and BRST approach that allow to carry out the renormalization procedure in a relativistic invariant fashion to obtain unitary renormalized S-matrix. Often, the unitary S-matrix is not obtained by using a priori prescribed computation procedure. This is to say that the unitary S-matrix is often obtained by using the self-consistent computational framework. In some sense one can say that it is the procedure for the derivation of the unitary S-matrix that provides us the constructive way for building theory of quantized interacting fields.

At the present time, a notion of the S-matrix is more or less well understood for theories of quantized fields in flat space. Generalization of notion of the S-matrix to quantized fields in AdS space seems to be problematic. On the other hand, as was pointed out somewhere in earlier literature, the role of the S-matrix for fields in AdS space can be delegated to so called effective action of shadow fields. Such action is defined through a continual integral over AdS field with some particular boundary condition to be imposed on boundary value of AdS field. That particular boundary value of AdS field is referred to as shadow field. Therefore the continual integral provides the realization of the effective action in terms of the shadow field. In the framework of AdS/CFT correspondence, the effective action is realized as a generating function for correlation functions of CFT which, according to Maldacena conjecture, is dual to AdS theory. By analogy with the S-matrix in flat space, one can believe that the full quantum effective action of shadow fields provides the constructive way for a definition of quantum field theory in AdS space. For these reasons, study of the effective action of shadow fields seems to be well motivated.

In quadratic approximation, the effective actions for spin-1 and spin-2 shadow fields were studied in Refs.[3]-[6], while the effective action for arbitrary spin- s shadow field was considered in Refs.[7, 8]. Some group-theoretical issues related to problems of the computation of the effective actions were addressed in Refs.[9]-[12]. Effective action for arbitrary spin shadow field found in Refs.[7, 8] was presented in terms of gauge field subject to some differential constraints. In Sec. 3, we introduce shadow Faddeev-Popov fields and find BRST invariant effective action expressed in terms of fields which are not subject to any differential constraints.

Effective actions of shadow fields that are dual to massless AdS fields has logarithmic divergences. For the case of spin-2 field in Ref.[4] and arbitrary spin- s field in Ref.[7], it was demonstrated that the logarithmic divergences of the effective actions of spin-2 and arbitrary spin- s shadow fields turn out to be actions of the respective conformal spin-2 and arbitrary spin- s conformal fields. In Sec. 4, we show that logarithmic divergence of the BRST invariant action of arbitrary spin- s canonical shadow field turns out to be BRST invariant action of arbitrary spin- s conformal field. In due course, we demonstrate that Nakanishi-Laudrup fields entering BRST invariant action of conformal field can geometrically be interpreted as boundary values of massless AdS fields.

In recent time, some attention has been paid to the problem of the computation of one-loop partition function for AdS fields (see, e.g., Refs.[13, 14] and references therein). As BRST invariant Lagrangian for AdS fields provides systematical and self-contained way for the computation of partition function we start our discussion in Sec. 2 with building BRST invariant Lagrangian for AdS fields and apply such Lagrangian for the computation of partition function.

2 $so(d, 1)$ covariant approach and partition function

In this section, we review the Lagrangian $so(d, 1)$ covariant formulation of dynamics of free fields in $(A)dS_{d+1}$ space developed in Refs.[15, 16]. In Sec.2.1, we will use our approach for the derivation of BRST invariant Lagrangians of $(A)dS$ fields and apply such the Lagrangians for the study of partition functions.

Field content. For the gauge invariant description of spin- s massless field in $(A)dS_{d+1}$, we use a double-traceless tensor field of the $so(d, 1)$ algebra,

$$\phi^{A_1 \dots A_s}, \quad \phi^{AABBA_5 \dots A_s} = 0, \quad \text{for } s \geq 4, \quad (2.1)$$

while, for the gauge invariant description of spin- s massive field in AdS_{d+1} , we use the following scalar, vector, and totally symmetric double-traceless tensor fields of the $so(d, 1)$ algebra:

$$\phi^{A_1 \dots A_{s'}}, \quad s' = 0, 1, \dots, s, \quad \phi^{AABBA_5 \dots A_{s'}} = 0 \quad \text{for } s' \geq 4. \quad (2.2)$$

We note that field contents in (2.1), (2.2) enter so called metric-like formulation of massless and massive fields.¹ Also we note that the $so(d, 1)$ covariant formulation is developed by using an arbitrary parametrization of $(A)dS$ space.

The presentation of gauge-invariant Lagrangian can considerably be simplified by using generating form of gauge fields. To this end we introduce oscillators α^A, ζ . Using such oscillators, gauge fields in (2.1), (2.2) can be collected into a ket-vector $|\phi\rangle$ as follows

$$|\phi\rangle = \frac{1}{s!} \alpha^{A_1} \dots \alpha^{A_s} \phi^{A_1 \dots A_s} |0\rangle, \quad \text{for massless field;} \quad (2.3)$$

$$|\phi\rangle = \sum_{s'=0}^s \frac{\zeta^{s-s'}}{\sqrt{(s-s')!}} |\phi^{s'}\rangle,$$

$$|\phi^{s'}\rangle = \frac{1}{s'!} \alpha^{A_1} \dots \alpha^{A_{s'}} \phi^{A_1 \dots A_{s'}} |0\rangle, \quad \text{for massive field.} \quad (2.4)$$

Gauge invariant Lagrangian. In terms of the ket-vector $|\phi\rangle$, gauge invariant Lagrangians for massless and massive fields can be presented on an equal footing as follows,

$$\mathcal{L} = \frac{1}{2} e \langle \phi | E | \phi \rangle, \quad (2.5)$$

$$E = \mu (\square_{(A)dS} + m_1 + \rho \alpha^2 \bar{\alpha}^2) - L \bar{L}, \quad (2.6)$$

$$\bar{L} \equiv \bar{\alpha} D - \frac{1}{2} \alpha D \bar{\alpha}^2 - \bar{e}_1 \Pi^{[1,2]} + \frac{1}{2} e_1 \bar{\alpha}^2, \quad (2.7)$$

$$L \equiv \alpha D - \frac{1}{2} \alpha^2 \bar{\alpha} D - e_1 \Pi^{[1,2]} + \frac{1}{2} \bar{e}_1 \alpha^2, \quad (2.8)$$

where $\rho = \epsilon/R^2$, $\epsilon = 1(-1)$ for $dS(AdS)$, R is radius of $(A)dS$, $e = \det e_m^A$, e_m^A are vielbein in $(A)dS$, $\square_{(A)dS}$ D'Alembert operator in $(A)dS$, while m_1, e_1, \bar{e}_1 are given by

$$m_1 = \rho(s(s+d-5) - 2d+4), \quad e_1 = 0, \quad \bar{e}_1 = 0, \quad \text{for massless field,} \quad (2.9)$$

¹ In the framework of metric-like approach, massless fields in AdS_4 were studied in Ref.[17], while massless fields in AdS_{d+1} , $d \geq 3$, were considered in Refs.[10, 18]. For massive field, field content in (2.2), was introduced in [19]. In the framework of frame-like approach, massless and massive fields were studied in Refs.[20].

$$\begin{aligned}
\mathbf{m}_1 &= -m^2 + \rho \left(s(s+d-5) - 2d + 4 + N_\zeta(2s+d-1-N_\zeta) \right), \\
\mathbf{e}_1 &= \zeta \mathbf{e}_\zeta, \quad \bar{\mathbf{e}}_1 = -\mathbf{e}_\zeta \bar{\zeta}, \\
\mathbf{e}_\zeta &\equiv \left(\frac{2s+d-3-N_\zeta}{2s+d-3-2N_\zeta} (m^2 - \rho N_\zeta(2s+d-4-N_\zeta)) \right)^{1/2}, \text{ for massive field.} \quad (2.10)
\end{aligned}$$

Operators $\alpha \mathbf{D}$, α^2 , $\boldsymbol{\mu}$, $\Pi^{[1,2]}$, N_ζ appearing in (2.6)-(2.10) are given in Appendix. Our bra-vectors are defined as $\langle \phi | \equiv (|\phi \rangle)^\dagger$. We emphasize that, in our approach, it is the use of the operators \mathbf{L} , $\bar{\mathbf{L}}$ (2.7), (2.8) that simplifies considerably the structure of Lagrangian.² Also we note the helpful relation $e\langle \phi | \mathbf{L} \bar{\mathbf{L}} | \phi \rangle = -e\langle \bar{\mathbf{L}} \phi | \bar{\mathbf{L}} | \phi \rangle$ which is valid up to total derivatives. Note that $\langle \bar{\mathbf{L}} \phi | \equiv (\bar{\mathbf{L}} | \phi \rangle)^\dagger$.

Taking into account that the tensor fields entering our ket-vector $|\phi \rangle$ are double-traceless, $(\bar{\alpha}^2)^2 |\phi \rangle = 0$, we note that, alternatively, operator E (2.6) can be represented as

$$E = \square_{(A)dS} + M_1 - \frac{1}{4} \alpha^2 \bar{\alpha}^2 (\square_{(A)dS} + M_2) - \mathbf{L} \bar{\mathbf{L}}, \quad (2.11)$$

where M_1, M_2 are given by

$$\begin{aligned}
M_1 &= \rho(s(s+d-5) - 2d + 4), \\
M_2 &= \rho(s(s+d-1) - 6), \quad \text{for massless field;} \quad (2.12)
\end{aligned}$$

$$\begin{aligned}
M_1 &= -m^2 + \rho \left(s(s+d-5) - 2d + 4 + N_\zeta(2s+d-1-N_\zeta) \right), \\
M_2 &= -m^2 + \rho \left(s(s+d-1) - 6 + N_\zeta(2s+d-5-N_\zeta) \right), \quad \text{for massive field.} \quad (2.13)
\end{aligned}$$

Gauge symmetries. For the description of gauge symmetries of the massless spin- s field, we use totally symmetric traceless rank- $(s-1)$ tensor field of the $so(d, 1)$ algebra,

$$\xi^{A_1 \dots A_{s-1}}, \quad \xi^{AA_3 \dots A_{s-1}} = 0, \quad \text{for } s \geq 3, \quad (2.14)$$

while, for the description of gauge symmetries of the massive spin- s field, we use the following scalar, vector, and totally symmetric traceless tensor fields of the $so(d, 1)$ algebra:

$$\xi^{A_1 \dots A_{s'}}, \quad s' = 0, 1, \dots, s-1, \quad \xi^{AA_3 \dots A_{s'}} = 0 \quad \text{for } s' \geq 2. \quad (2.15)$$

The presentation of gauge transformations can considerably be simplified by using the generating form of the scalar, vector, and tensor fields in (2.14), (2.15),

$$|\xi \rangle = \frac{1}{(s-1)!} \alpha^{A_1} \dots \alpha^{A_{s-1}} \xi^{A_1 \dots A_{s-1}} |0 \rangle, \quad \text{for massless field;} \quad (2.16)$$

$$\begin{aligned}
|\xi \rangle &= \sum_{s'=0}^{s-1} \frac{\zeta^{s-1-s'}}{\sqrt{(s-1-s')!}} |\xi^{s'} \rangle, \\
|\xi^{s'} \rangle &= \frac{1}{s'!} \alpha^{A_1} \dots \alpha^{A_{s'}} \xi^{A_1 \dots A_{s'}} |0 \rangle, \quad \text{for massive field.} \quad (2.17)
\end{aligned}$$

² Representation for Lagrangian in (2.5)-(2.8) was found in Refs.[15, 16]. Other representations for the Lagrangian may be found in Refs.[18, 19].

In terms of the ket-vectors $|\phi\rangle, |\xi\rangle$, gauge transformations of the massless and massive fields can be presented on an equal footing as follows

$$\delta|\phi\rangle = \mathbf{G}|\xi\rangle, \quad \mathbf{G} \equiv \alpha\mathbf{D} - \mathbf{e}_1 - \alpha^2 \frac{1}{2N_\alpha + d - 1} \bar{\mathbf{e}}_1, \quad (2.18)$$

where operators $\mathbf{e}_1, \bar{\mathbf{e}}_1$ are given in (2.9), (2.10), while N_α is defined in Appendix.

2.1 BRST invariant Lagrangian and partition function

BRST invariant Lagrangian. To built BRST invariant Lagrangian one needs to introduce Faddeev-Popov and Nakanishi-Laudrup fields.³ Generating form of Faddeev-Popov fields is described by ket-vectors $|c\rangle, |\bar{c}\rangle$, while a generating form of Nakanishi-Laudrup fields is described by ket-vector $|b\rangle$. Decomposition of these ket-vectors into scalar, vector, and tensor fields of the $so(d, 1)$ algebra takes the form

$$\begin{aligned} |c\rangle &= \frac{1}{(s-1)!} \alpha^{A_1} \dots \alpha^{A_{s-1}} c^{A_1 \dots A_{s-1}} |0\rangle, & |\bar{c}\rangle &= \frac{1}{(s-1)!} \alpha^{A_1} \dots \alpha^{A_{s-1}} \bar{c}^{A_1 \dots A_{s-1}} |0\rangle, \\ |b\rangle &= \frac{1}{(s-1)!} \alpha^{A_1} \dots \alpha^{A_{s-1}} b^{A_1 \dots A_{s-1}} |0\rangle, & & \text{for massless field;} \end{aligned} \quad (2.19)$$

$$\begin{aligned} |c\rangle &= \sum_{s'=0}^{s-1} \frac{\zeta^{s-1-s'}}{\sqrt{(s-1-s')!}} |c^{s'}\rangle, & |c^{s'}\rangle &= \frac{1}{s'!} \alpha^{A_1} \dots \alpha^{A_{s'}} c^{A_1 \dots A_{s'}} |0\rangle, \\ |\bar{c}\rangle &= \sum_{s'=0}^{s-1} \frac{\zeta^{s-1-s'}}{\sqrt{(s-1-s')!}} |\bar{c}^{s'}\rangle, & |\bar{c}^{s'}\rangle &= \frac{1}{s'!} \alpha^{A_1} \dots \alpha^{A_{s'}} \bar{c}^{A_1 \dots A_{s'}} |0\rangle, \\ |b\rangle &= \sum_{s'=0}^{s-1} \frac{\zeta^{s-1-s'}}{\sqrt{(s-1-s')!}} |b^{s'}\rangle, & |b^{s'}\rangle &= \frac{1}{s'!} \alpha^{A_1} \dots \alpha^{A_{s'}} b^{A_1 \dots A_{s'}} |0\rangle, \\ & & & \text{for massive field.} \end{aligned} \quad (2.20)$$

Tensor fields (2.19), (2.20) are totally symmetric traceless tensor fields of the $so(d, 1)$ algebra.

Using the ket-vectors, the BRST invariant Lagrangian \mathcal{L}_{tot} in arbitrary α -gauge can be presented as

$$\mathcal{L}_{\text{tot}} = \mathcal{L} + \mathcal{L}_{\text{qu}}, \quad (2.21)$$

$$\frac{1}{e} \mathcal{L}_{\text{qu}} = -\langle b | \bar{\mathbf{L}} | \phi \rangle + \langle \bar{c} | (\square_{(A)} \text{dS} + M_{\text{FP}}) | c \rangle + \frac{1}{2} \alpha \langle b | b \rangle, \quad (2.22)$$

$$M_{\text{FP}} \equiv -m^2 + \rho \left((s-1)(s+d-2) + N_\zeta(2s+d-3-N_\zeta) \right), \quad (2.23)$$

where operator $\bar{\mathbf{L}}$ is defined in (2.7). Lagrangian (2.21) is invariant under the following BRST and anti-BRST transformations (invariance of Lagrangian is assumed up to total derivative)

³ In this paper, we deal with global BRST symmetries. Study of massless and massive fields with local BRST symmetries may be found in Refs.[21]-[30].

$$s|\phi\rangle = \mathbf{G}|c\rangle, \quad s|c\rangle = 0, \quad s|\bar{c}\rangle = |b\rangle, \quad s|b\rangle = 0, \quad (2.24)$$

$$\bar{s}|\phi\rangle = \mathbf{G}|\bar{c}\rangle, \quad \bar{s}|c\rangle = -|b\rangle, \quad \bar{s}|\bar{c}\rangle = 0, \quad \bar{s}|b\rangle = 0, \quad (2.25)$$

where \mathbf{G} is given in (2.18). BRST and anti-BRST transformations (2.24), (2.25) are off-shell nilpotent: $s^2 = 0, \bar{s}^2 = 0, s\bar{s} + \bar{s}s = 0$.

Partition function. For the computation of a partition function, we chose the $\alpha = 1$ gauge and integrate out Nakanishi-Laudrup fields. Doing so, we cast Lagrangian (2.21) into the form

$$\frac{1}{e}\mathcal{L}_{\text{tot}} = \frac{1}{2}\langle\phi|\mu(\square_{(A)\text{dS}} + \mathbf{m}_1 + \rho\alpha^2\bar{\alpha}^2)|\phi\rangle + \langle\bar{c}|(\square_{(A)\text{dS}} + M_{\text{FP}})|c\rangle. \quad (2.26)$$

We recall that a partition function does not depend on a choice of gauge condition. However we would like to emphasize that it is the representation for Lagrangian in (2.5)-(2.8) and the $\alpha = 1$ gauge that simplify considerably the expression for \mathcal{L}_{tot} (2.26) and hence the computation of a partition function.⁴ Alternatively, Lagrangian (2.26) can be represented as

$$\frac{1}{e}\mathcal{L}_{\text{tot}} = \frac{1}{2}\langle\phi|\left(\square_{(A)\text{dS}} + M_1 - \frac{1}{4}\alpha^2\bar{\alpha}^2(\square_{(A)\text{dS}} + M_2)\right)|\phi\rangle + \langle\bar{c}|(\square + M_{\text{FP}})|c\rangle. \quad (2.27)$$

It is convenient to decompose the double-traceless ket-vector $|\phi\rangle$ into two traceless ket-vectors $|\phi_{\text{I}}\rangle, |\phi_{\text{II}}\rangle$ by using the relations

$$|\phi\rangle = |\phi_{\text{I}}\rangle + \alpha^2\mathcal{N}|\phi_{\text{II}}\rangle, \quad \bar{\alpha}^2|\phi_{\text{I}}\rangle = 0, \quad \bar{\alpha}^2|\phi_{\text{II}}\rangle = 0, \quad (2.28)$$

$$\mathcal{N} \equiv ((2s + d - 3)(2s + d - 5))^{-1/2}, \quad \text{for massless field}, \quad (2.29)$$

$$\mathcal{N} \equiv ((2s + d - 3 - 2N_\zeta)(2s + d - 5 - 2N_\zeta))^{-1/2}, \quad \text{for massive field}. \quad (2.30)$$

Plugging (2.28) into (2.27), we obtain

$$\frac{1}{e}\mathcal{L}_{\text{tot}} = \frac{1}{2}\langle\phi_{\text{I}}|(\square_{(A)\text{dS}} + M_1)|\phi_{\text{I}}\rangle - \frac{1}{2}\langle\phi_{\text{II}}|(\square_{(A)\text{dS}} + M_2 + 4\rho)|\phi_{\text{II}}\rangle + \langle\bar{c}|(\square_{(A)\text{dS}} + M_{\text{FP}})|c\rangle. \quad (2.31)$$

Decomposition of ket-vectors $|\phi_{\text{I}}\rangle, |\phi_{\text{II}}\rangle$ (2.28) into scalar, vector, and traceless tensor fields of the $so(d, 1)$ algebra is given by

$$|\phi_{\text{I}}\rangle = \frac{1}{s!}\alpha^{A_1}\dots\alpha^{A_s}\phi_{\text{I}}^{A_1\dots A_s}|0\rangle, \quad (2.32)$$

$$|\phi_{\text{II}}\rangle = \frac{1}{(s-2)!}\alpha^{A_1}\dots\alpha^{A_{s-2}}\phi_{\text{II}}^{A_1\dots A_{s-2}}|0\rangle, \quad \text{for massless field;}$$

$$|\phi_{\text{I}}\rangle = \sum_{s'=0}^s \frac{\zeta^{s-s'}}{\sqrt{(s-s')!}}|\phi_{\text{I}}^{s'}\rangle, \quad |\phi_{\text{II}}\rangle = \sum_{s'=0}^{s-2} \frac{\zeta^{s-2-s'}}{\sqrt{(s-2-s')!}}|\phi_{\text{II}}^{s'}\rangle,$$

$$|\phi_{\text{I,II}}^{s'}\rangle = \frac{1}{s'!}\alpha^{A_1}\dots\alpha^{A_{s'}}\phi_{\text{I,II}}^{A_1\dots A_{s'}}|0\rangle, \quad \text{for massive field}. \quad (2.33)$$

⁴ Recent research in Ref.[31] provides new interesting way for proving gauge independence of a partition function.

In terms of the scalar, vector, and tensor fields, Lagrangian (2.31) can be presented as

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{I}}^s - \mathcal{L}_{\text{II}}^{s-2} + \mathcal{L}_{\text{FP}}^{s-1}, \quad \text{for massless field,} \quad (2.34)$$

$$\mathcal{L}_{\text{tot}} = \sum_{s'=0}^s \mathcal{L}_{\text{I}}^{s'} - \sum_{s'=0}^{s-2} \mathcal{L}_{\text{II}}^{s'} + \sum_{s'=0}^{s-1} \mathcal{L}_{\text{FP}}^{s'}, \quad \text{for massive field,} \quad (2.35)$$

$$\mathcal{L}_{\text{I,II}}^{s'} = \frac{e}{2s'!} \phi_{\text{I,II}}^{A_1 \dots A_{s'}} (\mathcal{D}^2 + M_1^{s'}) \phi_{\text{I,II}}^{A_1 \dots A_{s'}}, \quad \mathcal{L}_{\text{FP}}^{s'} = \frac{e}{s'!} \bar{c}^{A_1 \dots A_{s'}} (\mathcal{D}^2 + M_1^{s'}) c^{A_1 \dots A_{s'}}, \quad (2.36)$$

$$M_1^s \equiv \rho((s-2)(s+d-2)-s), \quad M_1^{s-1} \equiv \rho(s-1)(s+d-2),$$

$$M_1^{s-2} \equiv \rho(s(s+d-1)-2), \quad \text{for massless field,} \quad (2.37)$$

$$M_1^{s'} \equiv -m^2 + \rho(2(s-1)(s+d-2) - s'(s'+d-1)), \quad \text{for massive field.} \quad (2.38)$$

From (2.34)-(2.38), we see that the partition function is given by

$$Z = D^{s-1} D^{s-1} / D^s D^{s-2}, \quad \text{for massless field,} \quad (2.39)$$

$$Z = \prod_{s'=0}^{s-1} D^{s'} D^{s'} / \prod_{s'=0}^s D^{s'} \prod_{s'=0}^{s-2} D^{s'}$$

$$= D^{s-1} / D^s, \quad \text{for massive field,} \quad (2.40)$$

$$D^{s'} \equiv (\det(-\mathcal{D}^2 - M_1^{s'}))^{1/2}, \quad (2.41)$$

where, in relation (2.41), the determinant of D'Alembert operator is evaluated for rank- s' traceless tensor field. Often, it is convenient to use the following relation for $D^{s'}$ (2.41):

$$D^{s'} = D^{s'\perp} D^{s'-1}, \quad (2.42)$$

where $D^{s'\perp}$ takes the same form as in (2.41), while the determinant of D'Alembert operator is evaluated on space of traceless and divergence-free rank- s' tensor field. Using (2.42) in (2.39),(2.40), we obtain

$$Z = D^{s-1\perp} / D^{s\perp} \quad \text{for massless field,} \quad (2.43)$$

$$Z = 1 / D^{s\perp} \quad \text{for massive field.} \quad (2.44)$$

In earlier literature, partition functions (2.43), (2.44) were obtained by different methods in Refs.[13, 14].

3 $so(d-1, 1)$ covariant approach and effective action

In this section, we review Lagrangian $so(d-1, 1)$ covariant approach to dynamics of free fields in AdS_{d+1} developed in Refs.[15, 16]. In Sec.3.1, we use our approach for the derivation of BRST invariant Lagrangians of AdS fields and apply such Lagrangians for derivation of BRST invariant effective action of shadow fields.

To develop $so(d-1, 1)$ covariant approach to fields in AdS_{d+1} we use the Poincaré parametrization of AdS_{d+1} ,

$$ds^2 = \frac{1}{z^2}(dx^a dx^a + dz dz). \quad (3.1)$$

From (3.1), we see that the line element respects manifest symmetries of the $so(d-1, 1)$ algebra. **Field content.** In the framework of metric-like approach, for the gauge invariant description of spin- s massless field in AdS_{d+1} , we use the following scalar, vector and totally symmetric double-traceless tensor field of the $so(d-1, 1)$ algebra,

$$\phi^{a_1 \dots a_{s'}}, \quad s' = 0, 1, \dots, s, \quad \phi^{aabb a_5 \dots a_{s'}} = 0, \quad \text{for } s' \geq 4, \quad (3.2)$$

while, for the gauge invariant description of spin- s massive field in AdS_{d+1} , we use the following scalar, vector and totally symmetric double-traceless tensor fields of the $so(d-1, 1)$ algebra:

$$\phi_\lambda^{a_1 \dots a_{s'}}, \quad s' = 0, 1, \dots, s, \quad \lambda \in [s-s']_2, \quad \phi_\lambda^{aabb a_5 \dots a_{s'}} = 0, \quad \text{for } s' \geq 4. \quad (3.3)$$

Here and below the notation $\lambda \in [n]_2$ implies that λ takes the following values: $\lambda = -n, -n+2, -n+4, \dots, n-4, n-2, n$.

To simplify the presentation of gauge gauge-invariant Lagrangian we use generating form of gauge fields. To this end we introduce the oscillators $\alpha^a, \alpha^z, \zeta$ and note that gauge fields in (3.2), (3.3) can be collected into a ket-vector $|\phi\rangle$ as follows

$$|\phi\rangle \equiv \sum_{s'=0}^s \frac{\alpha_z^{s-s'}}{\sqrt{(s-s')!}} |\phi^{s'}\rangle, \quad (3.4)$$

$$|\phi^{s'}\rangle \equiv \frac{1}{s'!} \alpha^{a_1} \dots \alpha^{a_{s'}} \phi^{a_1 \dots a_{s'}} |0\rangle, \quad \text{for massless field,}$$

$$|\phi\rangle = \sum_{s'=0}^s |\phi^{s'}\rangle, \quad (3.5)$$

$$|\phi^{s'}\rangle = \sum_{\lambda \in [s-s']_2} \frac{\zeta^{\frac{s-s'+\lambda}{2}} \alpha_z^{\frac{s-s'-\lambda}{2}} \alpha^{a_1} \dots \alpha^{a_{s'}}}{s'! \sqrt{(\frac{s-s'+\lambda}{2})! (\frac{s-s'-\lambda}{2})!}} \phi_\lambda^{a_1 \dots a_{s'}} |0\rangle, \quad \text{for massive field.}$$

Gauge invariant Lagrangian. In terms of ket-vectors $|\phi\rangle$ (3.4),(3.5), gauge invariant action and Lagrangian for massless and massive fields can be presented on an equal footing as follows

$$S = \int d^d x dz \mathcal{L}, \quad (3.6)$$

$$\mathcal{L} = \frac{1}{2} \langle \phi | \mu (\square - \mathcal{M}^2) | \phi \rangle + \frac{1}{2} \langle \bar{L} \phi | | \bar{L} \phi \rangle, \quad (3.7)$$

$$\bar{L} \equiv \bar{\alpha} \partial - \frac{1}{2} \alpha \partial \bar{\alpha}^2 - \bar{e}_1 \Pi^{[1,2]} + \frac{1}{2} e_1 \bar{\alpha}^2, \quad (3.8)$$

$\square \equiv \partial^a \partial^a$, $\partial^a = \eta^{ab} \partial / \partial x^b$, $|\bar{L} \phi\rangle \equiv \bar{L} |\phi\rangle$. Expressions for scalar products like $\alpha \partial, \alpha^2, \mu, \Pi^{[1,2]}$ are defined in Appendix. Bra-vectors $\langle \phi |, \langle \bar{L} \phi |$ are defined as follows, $\langle \phi | \equiv (|\phi\rangle)^\dagger$, $\langle \bar{L} \phi | \equiv (|\bar{L} \phi\rangle)^\dagger$. Operators $\mathcal{M}^2, e_1, \bar{e}_1$ are given in Table. From (3.7), we see that the kinetic terms of massless and

massive fields have one and the same dependence on the vector oscillators α^a and the derivatives ∂^a . From Table, we see that that all dependence of the kinetic terms of the massless and massive fields on the scalar oscillators ζ , α^z , the radial coordinate z , and the radial derivative $\partial_z = \partial/\partial z$ is described completely by the operators \mathcal{M}^2 , e_1 , \bar{e}_1 .

Gauge symmetries. For the discussion of gauge symmetries, we introduce gauge transformation parameters. Generating form of gauge transformation parameters is described by a ket-vector $|\xi\rangle$. Decomposition of the ket-vector into scalar, vector, and totally symmetric traceless tensor fields of the $so(d-1, 1)$ algebra is given by

$$|\xi\rangle \equiv \sum_{s'=0}^{s-1} \frac{\alpha_z^{s-1-s'}}{\sqrt{(s-1-s')!}} |\xi^{s'}\rangle, \quad (3.9)$$

$$|\xi^{s'}\rangle \equiv \frac{1}{s'!} \alpha^{a_1} \dots \alpha^{a_{s'}} \zeta^{a_1 \dots a_{s'}} |0\rangle, \quad \text{for massless field,}$$

$$|\xi\rangle = \sum_{s'=0}^{s-1} |\xi^{s'}\rangle,$$

$$|\xi^{s'}\rangle = \sum_{\lambda \in [s-1-s']_2} \frac{\zeta^{\frac{s-1-s'+\lambda}{2}} \alpha_z^{\frac{s-1-s'-\lambda}{2}} \alpha^{a_1} \dots \alpha^{a_{s'}}}{s'! \sqrt{(\frac{s-1-s'+\lambda}{2})! (\frac{s-1-s'-\lambda}{2})!}} \xi_\lambda^{a_1 \dots a_{s'}} |0\rangle, \quad \text{for massive field.} \quad (3.10)$$

In terms of the ket-vectors $|\phi\rangle$, $|\xi\rangle$ above discussed, gauge transformations of massless and massive fields can be presented on an equal footing,

$$\delta|\phi\rangle = G|\xi\rangle, \quad G \equiv \alpha\partial - e_1 - \alpha^2 \frac{1}{2N_\alpha + d - 2} \bar{e}_1, \quad (3.11)$$

where the operators e_1 , \bar{e}_1 are given in the Table, while N_α is defined in Appendix.

Table. Operators \mathcal{M}^2 , e_1 , \bar{e}_1 entering Lagrangian, gauge transformations and BRST transformations of AdS field in the framework of $so(d-1, 1)$ covariant formulation. m is mass parameter of massive field. In Table, we present also the operators e_1 , \bar{e}_1 entering BRST transformations canonical and anomalous shadows.

Fields	\mathcal{M}^2	e_1	\bar{e}_1
massless spin- s field in AdS_{d+1}	$-\partial_z^2 + \frac{1}{z^2}(\nu^2 - \frac{1}{4})$	$-\alpha^z e_z \mathcal{T}_{\nu-\frac{1}{2}}$	$-\mathcal{T}_{-\nu+\frac{1}{2}} e_z \bar{\alpha}^z$
	$\nu \equiv s + \frac{d-4}{2} - N_z, \quad \mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z} \quad \partial_z \equiv \partial/\partial z$		
massive spin- s field in AdS_{d+1}	$-\partial_z^2 + \frac{1}{z^2}(\nu^2 - \frac{1}{4})$	$-\zeta r_\zeta \mathcal{T}_{\nu-\frac{1}{2}} - \alpha^z r_z \mathcal{T}_{\nu-\frac{1}{2}}$	$-\mathcal{T}_{\nu+\frac{1}{2}} r_\zeta \bar{\zeta} - \mathcal{T}_{-\nu+\frac{1}{2}} r_z \bar{\alpha}^z$
	$\nu \equiv \kappa + N_\zeta - N_z, \quad \mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z}, \quad \partial_z \equiv \partial/\partial z$		
canonical spin- s shadow in $R^{d-1,1}$	-	$\alpha^z e_z \square$	$-e_z \bar{\alpha}^z$
anomalous spin- s shadow in $R^{d-1,1}$	-	$\zeta r_\zeta + \alpha^z r_z \square$	$-r_\zeta \bar{\zeta} \square - r_z \bar{\alpha}^z$
$e_z = \left(\frac{2s+d-4-N_z}{2s+d-4-2N_z} \right)^{1/2}$ $r_\zeta = \left(\frac{(s+\frac{d-4}{2}-N_\zeta)(\kappa-s-\frac{d-4}{2}+N_\zeta)(\kappa+1+N_\zeta)}{2(s+\frac{d-4}{2}-N_\zeta-N_z)(\kappa+N_\zeta-N_z)(\kappa+N_\zeta-N_z+1)} \right)^{1/2}$ $r_z = \left(\frac{(s+\frac{d-4}{2}-N_z)(\kappa+s+\frac{d-4}{2}-N_z)(\kappa-1-N_z)}{2(s+\frac{d-4}{2}-N_\zeta-N_z)(\kappa+N_\zeta-N_z)(\kappa+N_\zeta-N_z-1)} \right)^{1/2}$ $\kappa \equiv \sqrt{m^2 + \left(s + \frac{d-4}{2} \right)^2}$			

3.1 BRST invariant Lagrangian of AdS fields and effective action of shadow fields

BRST invariant Lagrangian of AdS field. To built BRST invariant Lagrangian we should introduce Faddeev-Popov and Nakanishi-Laudrup fields. Generating form of Faddeev-Popov fields is described by ket-vectors $|c\rangle$, $|\bar{c}\rangle$, while generating form of Nakanishi-Laudrup fields is described by ket-vector $|b\rangle$. The decomposition of these ket-vectors into the corresponding scalar, vector, and traceless tensor fields $\bar{c}^{a_1 \dots a_{s'}}$, $c^{a_1 \dots a_{s'}}$, $b^{a_1 \dots a_{s'}}$ takes the same form as the one in (3.9), (3.10) and can be obtained by using the following substitutions in (3.9), (3.10),

$$\xi \rightarrow c, \quad \bar{\xi} \rightarrow \bar{c}, \quad \xi \rightarrow b. \quad (3.12)$$

By definition, tensorial fields entering the ket-vectors $|c\rangle$, $|\bar{c}\rangle$, $|b\rangle$ are totally symmetric traceless tensor fields of the $so(d-1, 1)$ algebra.

In terms of the ket-vectors, BRST invariant Lagrangian \mathcal{L}_{tot} in arbitrary α -gauge can be presented as

$$\mathcal{L}_{\text{tot}} = \mathcal{L} + \mathcal{L}_{\text{qu}}, \quad \mathcal{L}_{\text{qu}} = -\langle b|\bar{L}|\phi\rangle + \langle \bar{c}|(\square - \mathcal{M}^2)|c\rangle + \frac{1}{2}\alpha\langle b|b\rangle, \quad (3.13)$$

where operator \bar{L} is given in (3.8), while the operator \mathcal{M}^2 is defined in Table. Lagrangian (3.13) is invariant under the following BRST and anti-BRST transformations:

$$s|\phi\rangle = G|c\rangle, \quad s|c\rangle = 0, \quad s|\bar{c}\rangle = |b\rangle, \quad s|b\rangle = 0, \quad (3.14)$$

$$\bar{s}|\phi\rangle = G|\bar{c}\rangle, \quad \bar{s}|c\rangle = -|b\rangle, \quad \bar{s}|\bar{c}\rangle = 0, \quad \bar{s}|b\rangle = 0, \quad (3.15)$$

where the operator G is given in (3.11). BRST and anti-BRST transformations (3.14), (3.15) are off-shell nilpotent: $s^2 = 0$, $\bar{s}^2 = 0$, $s\bar{s} + \bar{s}s = 0$.

For the computation of effective action of shadow fields, we choose the $\alpha = 1$ gauge and integrate out the Nakanishi-Laudrup fields. Doing so, we cast the BRST invariant Lagrangian (3.13) into the form

$$\mathcal{L}_{\text{tot}} = \frac{1}{2}\langle\phi|\mu(\square - \mathcal{M}^2)|\phi\rangle + \langle\bar{c}|(\square - \mathcal{M}^2)|c\rangle. \quad (3.16)$$

BRST and anti-BRST symmetries of Lagrangian (3.16) are realized by the following transformations:

$$s|\phi\rangle = G|c\rangle, \quad s|c\rangle = 0, \quad s|\bar{c}\rangle = \bar{L}|\phi\rangle, \quad (3.17)$$

$$\bar{s}|\phi\rangle = G|\bar{c}\rangle, \quad \bar{s}|c\rangle = -\bar{L}|\phi\rangle, \quad \bar{s}|\bar{c}\rangle = 0, \quad (3.18)$$

where the operators \bar{L} and G are given in (3.8) and (3.11) respectively. Transformations (3.17), (3.18) are nilpotent only for on-shell Faddeev-Popov fields.

AdS/CFT correspondence. AdS/CFT correspondence is realized in two steps, at least. First, we solve equations of motion for AdS field with a suitable boundary conditions, i.e., we solve the Dirichlet problem. Boundary conditions are fixed by requiring the boundary value of AdS field to be related to some particular representation of conformal algebra which is referred to as shadow field. Namely the boundary values of massless AdS field and massive AdS field correspond to the representations of the conformal algebra which are referred to as canonical shadow field and anomalous shadow field respectively. Second, we plug solution of the Dirichlet problem into action of AdS field. Action of AdS field evaluated on the solution of the Dirichlet problem is referred to as effective action. For massless field, the effective action is functional of canonical shadow, while for massive field, the effective action is functional of anomalous shadow. We recall that, for free AdS field, kernel of effective action is a 2-point correlation function of CFT. Note that in our approach we solve the Dirichlet problem not only for gauge fields but also for Faddeev-Popov fields, i.e., we introduce boundary Faddeev-Popov shadow fields. This leads to BRST invariant effective action. We now describe details of the computation of the BRST invariant effective action.

Equations of motion for gauge fields and Faddeev-Popov fields obtained from Lagrangian (3.16) take the form

$$\square_\nu|\phi\rangle = 0, \quad \square_\nu|\bar{c}\rangle = 0, \quad \square_\nu|c\rangle = 0, \quad \square_\nu \equiv \square + \partial_z^2 - \frac{1}{z^2}(\nu^2 - \frac{1}{4}). \quad (3.19)$$

Solution to the Dirichlet problem for equations of motion (3.19) with boundary conditions for the gauge field $|\phi\rangle$ and the Faddeev-Popov fields $|c\rangle$, $|\bar{c}\rangle$ corresponding to the respective shadow

gauge fields, denoted by $|\phi_{\text{sh}}\rangle$, and shadow Faddeev-Popov fields, denoted by $|c_{\text{sh}}\rangle$, $|\bar{c}_{\text{sh}}\rangle$, can be presented as

$$|\phi(x, z)\rangle = \sigma_\nu \int d^d y G_\nu(x - y, z) |\phi_{\text{sh}}(y)\rangle, \quad (3.20)$$

$$|c(x, z)\rangle = \sigma_\nu \int d^d y G_\nu(x - y, z) |c_{\text{sh}}(y)\rangle, \quad (3.21)$$

$$|\bar{c}(x, z)\rangle = \sigma_\nu \int d^d y G_\nu(x - y, z) |\bar{c}_{\text{sh}}(y)\rangle, \quad (3.22)$$

$$\sigma_\nu \equiv \frac{2^\nu \Gamma(\nu)}{2^{\bar{\kappa}} \Gamma(\bar{\kappa})} (-)^{N_z}, \quad (3.23)$$

where the Green function G_ν is given by

$$G_\nu(x, z) = \frac{c_\nu z^{\nu+\frac{1}{2}}}{(|x|^2 + z^2)^{\nu+\frac{d}{2}}}, \quad c_\nu \equiv \frac{\Gamma(\nu + \frac{d}{2})}{\pi^{d/2} \Gamma(\nu)}, \quad (3.24)$$

Note that, for massless fields, relations in (3.20)-(3.22) provide solution of the Dirichlet problem with the canonical shadows $|\phi_{\text{sh}}\rangle$, $|c_{\text{sh}}\rangle$, $|\bar{c}_{\text{sh}}\rangle$ as boundary data, while, for massive fields, relations in (3.20)-(3.22) provide solution of the Dirichlet problem with the anomalous shadows $|\phi_{\text{sh}}\rangle$, $|c_{\text{sh}}\rangle$, $|\bar{c}_{\text{sh}}\rangle$ as boundary data. Also note that, the decomposition of $|\phi_{\text{sh}}\rangle$ into scalar, vector, and double-traceless tensor fields of the $so(d-1, 1)$ algebra takes the same form as in (3.4),(3.5), while the decomposition of $|c_{\text{sh}}\rangle$, $|\bar{c}_{\text{sh}}\rangle$, into scalar, vector, and traceless tensor fields of the $so(d-1, 1)$ algebra takes the same form as in (3.9),(3.10). The ν and $\bar{\kappa}$ appearing in (3.20)-(3.24) are given by

$$\nu \equiv s + \frac{d-4}{2} - N_z, \quad \bar{\kappa} \equiv s + \frac{d-4}{2}, \quad \text{for canonical shadows}, \quad (3.25)$$

$$\nu \equiv \kappa + N_\zeta - N_z, \quad \bar{\kappa} \equiv \kappa, \quad \text{for anomalous shadows}, \quad (3.26)$$

where κ is given in Table.

Taking into account (3.20)-(3.22), and the asymptotic behavior of the Green function

$$G_\nu(x, z) \xrightarrow{z \rightarrow 0} z^{-\nu+\frac{1}{2}} \delta^d(x), \quad (3.27)$$

we get the asymptotic behavior of solution of the Dirichlet problem for the gauge fields

$$|\phi(x, z)\rangle \xrightarrow{z \rightarrow 0} z^{-\nu+\frac{1}{2}} \sigma_\nu |\phi_{\text{sh}}(x)\rangle \quad (3.28)$$

and similar asymptotic behavior for the solution of Faddeev-Popov fields.

To find the effective action, we should plug our solution (3.20)-(3.22) into action (3.6) with Lagrangian (3.16). Note also that we should add to the action an appropriate boundary term. Using general method for finding boundary term in Ref.[32], we make sure that Lagrangian which involves a contribution of the boundary term can be presented as

$$\mathcal{L}_{\text{tot}} = \frac{1}{2} \langle \partial^a \phi | \mu | \partial^a \phi \rangle + \frac{1}{2} \langle \mathcal{T}_{\nu-\frac{1}{2}} \phi | \mu | \mathcal{T}_{\nu-\frac{1}{2}} \phi \rangle + \langle \partial^a \bar{c} | | \partial^a c \rangle + \langle \mathcal{T}_{\nu-\frac{1}{2}} \bar{c} | | \mathcal{T}_{\nu-\frac{1}{2}} c \rangle, \quad (3.29)$$

where \mathcal{T}_ν is defined in the Table. Note that, in order to adapt our formulas to Euclidean signature, we change sign of Lagrangian, $\mathcal{L} \rightarrow -\mathcal{L}$, when passing from (3.16) to (3.29). It is easy to verify that action (3.6), (3.29) considered on the solution of equations of motion can be represented as⁵

$$-S_{\text{eff}}^{\text{tot}} = \int d^d x \mathcal{L}_{\text{eff}}^{\text{tot}} \Big|_{z \rightarrow 0}, \quad \mathcal{L}_{\text{eff}}^{\text{tot}} = \frac{1}{2} \langle \phi | \mu \mathcal{T}_{\nu-\frac{1}{2}} | \phi \rangle + \langle \bar{c} | \mathcal{T}_{\nu-\frac{1}{2}} | c \rangle. \quad (3.30)$$

BRST invariant effective action of shadow field. Plugging (3.20)-(3.22) into (3.30), we get the following effective action:

$$-S_{\text{eff}}^{\text{tot}} = 2\bar{\kappa} c_{\bar{\kappa}} \Gamma_{\text{tot}}, \quad (3.31)$$

where Γ_{tot} is given by

$$\Gamma_{\text{tot}} \equiv \int d^d x_1 d^d x_2 \mathcal{L}_{12}^{\text{tot}}, \quad (3.32)$$

$$\mathcal{L}_{12}^{\text{tot}} \equiv \frac{1}{2} \langle \phi_{\text{sh}}(x_1) | \frac{\mu f_\nu}{|x_{12}|^{2\nu+d}} | \phi_{\text{sh}}(x_2) \rangle + \langle \bar{c}_{\text{sh}}(x_1) | \frac{f_\nu}{|x_{12}|^{2\nu+d}} | c_{\text{sh}}(x_2) \rangle, \quad (3.33)$$

$$f_\nu \equiv \frac{\Gamma(\nu + \frac{d}{2}) \Gamma(\nu + 1)}{4^{\bar{\kappa}-\nu} \Gamma(\bar{\kappa} + \frac{d}{2}) \Gamma(\bar{\kappa} + 1)}, \quad (3.34)$$

$$|x_{12}|^2 \equiv x_{12}^a x_{12}^a, \quad x_{12}^a = x_1^a - x_2^a, \quad (3.35)$$

and the operators N_ζ , N_z , μ are defined in Appendix. Recall that c_κ is given in (3.24), while the parameter κ is defined in Table.

Effective action (3.31) is invariant under the following BRST and anti-BRST transformations:

$$s|\phi_{\text{sh}}\rangle = G|c_{\text{sh}}\rangle, \quad s|c_{\text{sh}}\rangle = 0, \quad s|\bar{c}_{\text{sh}}\rangle = \bar{L}|\phi_{\text{sh}}\rangle, \quad (3.36)$$

$$\bar{s}|\phi_{\text{sh}}\rangle = G|\bar{c}_{\text{sh}}\rangle, \quad \bar{s}|c_{\text{sh}}\rangle = -\bar{L}|\phi_{\text{sh}}\rangle, \quad \bar{s}|\bar{c}_{\text{sh}}\rangle = 0, \quad (3.37)$$

$$G \equiv \alpha \partial - e_1 - \alpha^2 \frac{1}{2N_\alpha + d - 2} \bar{e}_1, \quad (3.38)$$

$$\bar{L} \equiv \bar{\alpha} \partial - \frac{1}{2} \alpha \partial \bar{\alpha}^2 - \bar{e}_1 \Pi^{[1,2]} + \frac{1}{2} e_1 \bar{\alpha}^2, \quad (3.39)$$

where operators e_1 , \bar{e}_1 corresponding to the canonical and anomalous shadows are given in Table. BRST and anti-BRST transformations given in (3.36), (3.37) are nilpotent.

4 BRST invariant Lagrangian of conformal fields

For canonical shadows, a kernel of effective action (3.32) is not well defined when d is even integer (see, e.g., [34]). Hopefully, the kernel becomes well defined upon using a dimensional regularization. When removing the regularization, we are left with a logarithmic divergence of the kernel. Below, we demonstrate that the logarithmic divergence of the BRST invariant effective action turns out to be BRST invariant action of a conformal field.

⁵ For the discussion of AdS/CFT correspondence, we use a Lagrangian approach. The study of AdS/CFT correspondence by using the higher-spin symmetries may be found in Refs.[33].

Using the notation $[d]$ for integer part of d , we introduce the regularization parameter ε by the relation

$$d - [d] = -2\varepsilon, \quad [d] - \text{even integer}. \quad (4.1)$$

Using (4.1) and taking into account the dependence of ν on d in (3.25), we note the following textbook asymptotic behavior for the kernel:

$$\frac{1}{|x|^{2\nu+d}} \stackrel{\varepsilon \sim 0}{\sim} \frac{1}{\varepsilon} \varrho_\nu \square^\nu \delta^{(d)}(x), \quad \varrho_\nu \equiv \frac{\pi^{d/2}}{4^\nu \Gamma(\nu+1) \Gamma(\nu + \frac{d}{2})}. \quad (4.2)$$

Plugging (4.2) into expression for Γ_{tot} in (3.32), we obtain

$$\begin{aligned} \Gamma_{\text{tot}} &\stackrel{\varepsilon \sim 0}{\sim} \frac{1}{\varepsilon} \varrho_{\nu_s} \int d^d x \mathcal{L}_{\text{tot}}, \quad \text{for canonical shadows,} \\ \mathcal{L}_{\text{tot}} &= \frac{1}{2} \langle \phi | \mu \square^\nu | \phi \rangle + \langle \bar{c} | \square^\nu | c \rangle, \quad \nu \equiv s + \frac{d-4}{2} - N_z. \end{aligned} \quad (4.3)$$

Note that, in order to simplify the notation, we make the identifications of the ket-vectors, $|\phi\rangle \equiv |\phi_{\text{sh}}\rangle$, $|c\rangle \equiv |c_{\text{sh}}\rangle$, $|\bar{c}\rangle \equiv |\bar{c}_{\text{sh}}\rangle$, when passing from (3.33) to (4.3). Lagrangian (4.3) is BRST invariant Lagrangian of spin- s conformal field.⁶ For the illustration purposes, we consider the Lagrangian for spin-1, spin-2, and arbitrary spin- s fields in turn.

4.1 Spin-1 conformal field

For spin-1 conformal field, Lagrangian (4.3) takes the form

$$\mathcal{L}_{\text{tot}} = \frac{1}{2} \phi^a \square^{k+1} \phi^a + \frac{1}{2} \phi \square^k \phi + \bar{c} \square^{k+1} c, \quad k \equiv \frac{d-4}{2}. \quad (4.4)$$

Lagrangian (4.4) is invariant under BRST and anti-BRST transformations given by

$$s\phi^a = \partial^a c, \quad s\phi = -\square c, \quad sc = 0, \quad s\bar{c} = \partial^a \phi^a + \phi, \quad (4.5)$$

$$\bar{s}\phi^a = \partial^a \bar{c}, \quad \bar{s}\phi = -\square \bar{c}, \quad \bar{s}c = -\partial^a \phi^a - \phi, \quad \bar{s}\bar{c} = 0. \quad (4.6)$$

It is easy to verify that transformations (4.5),(4.6) are off-shell nilpotent. This is related to the fact that the scalar field ϕ can be realized as the Nakanishi-Laudrup field. To see this we introduce a new field b by the following relation

$$b = \phi + \partial^a \phi^a. \quad (4.7)$$

Plugging $\phi = b - \partial^a \phi^a$ into (4.4) we cast Lagrangian (4.4) into the form

$$\mathcal{L}_{\text{tot}} = \mathcal{L} + \mathcal{L}_{\text{qu}}, \quad (4.8)$$

$$\mathcal{L} = -\frac{1}{4} F^{ab} \square^k F^{ab}, \quad F^{ab} \equiv \partial^a \phi^b - \partial^b \phi^a, \quad (4.9)$$

$$\mathcal{L}_{\text{qu}} = -b \square^k \partial^a \phi^a + \frac{1}{2} b \square^k b + \bar{c} \square^{k+1} c, \quad (4.10)$$

⁶ In this paper, we deal with global BRST transformations of free conformal fields. A study of free conformal fields with local BRST symmetries may be found in Refs.[35]. Discussion of general structure of interacting conformal fields in $3d$ may be found in Ref.[36].

while BRST and anti-BRST transformations (4.5), (4.6) can be cast into the form

$$s\phi^a = \partial^a c, \quad sb = 0, \quad sc = 0, \quad s\bar{c} = b, \quad (4.11)$$

$$\bar{s}\phi^a = \partial^a \bar{c}, \quad \bar{s}b = 0, \quad \bar{s}c = -b, \quad \bar{s}\bar{c} = 0. \quad (4.12)$$

From (4.10), (4.11), (4.12), we see that the field b can really be considered as the Nakanishi-Laudrup field. Note also that, only for $d = 4$, the field b can be excluded from the Lagrangian by using equations of motion for b .

In the framework of AdS/CFT correspondence, the fields ϕ^a, ϕ appear as boundary values of non-normalizable solution of equations of motion for spin-1 massless AdS field. This is the reason why we think that relation (4.7) can be considered as geometrical interpretation of the Nakanishi-Laudrup field.

4.2 Spin-2 conformal field

For spin-2 conformal field, Lagrangian (4.3) takes the form

$$\begin{aligned} \mathcal{L}_{\text{tot}} = & \frac{1}{4}\phi^{ab}\square^{k+1}\phi^{ab} - \frac{1}{8}\phi^{aa}\square^{k+1}\phi^{bb} + \frac{1}{2}\phi^a\square^k\phi^a + \frac{1}{2}\phi\square^{k-1}\phi \\ & + \bar{c}^a\square^{k+1}c^a + \bar{c}\square^k c, \quad k \equiv \frac{d-2}{2}. \end{aligned} \quad (4.13)$$

Lagrangian (4.13) is invariant under the BRST transformations

$$s\phi^{ab} = \partial^a c^b + \partial^b c^a + \frac{2}{d-2}\eta^{ab}c, \quad (4.14)$$

$$s\phi^a = \partial^a c - \square c^a, \quad (4.15)$$

$$s\phi = -u\square c, \quad (4.16)$$

$$sc^a = 0, \quad sc = 0, \quad (4.17)$$

$$s\bar{c}^a = \partial^b \phi^{ab} - \frac{1}{2}\partial^a \phi^{bb} + \phi^a, \quad (4.18)$$

$$s\bar{c} = \partial^a \phi^a + \frac{1}{2}\square\phi^{aa} + u\phi, \quad u \equiv \left(2\frac{d-1}{d-2}\right)^{1/2}. \quad (4.19)$$

The anti-BRST transformations are obtained from (4.13)-(4.19) by using the substitution $s \rightarrow \bar{s}$ and the following substitutions for all gauge fields and Faddeev-Popov fields: $\phi \rightarrow \phi, c \rightarrow \bar{c}, \bar{c} \rightarrow -c$.

It is easy to verify that BRST transformations (4.13)-(4.19) are off-shell nilpotent. This is related to the fact that the vector and scalar fields ϕ^a, ϕ can be realized as the Nakanishi-Laudrup field. To see this we introduce, in place of the field ϕ^a, ϕ , new fields b^a, b by the following relations

$$b^a \equiv \partial^b \phi^{ab} - \frac{1}{2}\partial^a \phi^{bb} + \phi^a, \quad b \equiv \partial^a \phi^a + \frac{1}{2}\square\phi^{aa} + u\phi. \quad (4.20)$$

Using (4.20) we get

$$\phi^a = b^a - \partial^b \phi^{ab} + \frac{1}{2}\partial^a \phi^{bb}, \quad u\phi = b - \partial^a b^a + \partial^a \partial^b \phi^{ab} - \square\phi^{aa}. \quad (4.21)$$

Plugging ϕ^a, ϕ (4.21) into (4.13), we cast Lagrangian (4.13) into the following form:

$$\mathcal{L}_{\text{tot}} = \mathcal{L} + \mathcal{L}_{\text{qu}}, \quad (4.22)$$

$$\mathcal{L} = R_{\text{lin}}^{ab} \square^{k-1} R_{\text{lin}}^{ab} - \frac{d}{4(d-1)} R_{\text{lin}} \square^{k-1} R_{\text{lin}}, \quad (4.23)$$

$$\begin{aligned} \mathcal{L}_{\text{qu}} = & -b^a \square^k (\partial^b \phi^{ab} - \frac{1}{2} \partial^a \phi^{bb}) + \frac{1}{u^2} (b - \partial^a b^a) \square^{k-1} (\partial^c \partial^e \phi^{ce} - \square \phi^{cc}) \\ & + \frac{1}{2} b^a \square^k b^a + \frac{1}{2u^2} (b - \partial^a b^a) \square^{k-1} (b - \partial^c b^c) + \bar{c}^a \square^{k+1} c^a + \bar{c} \square^k c, \end{aligned} \quad (4.24)$$

where $R_{\text{lin}}^{ab}, R_{\text{lin}}$ stand for the respective linearized Ricchi tensor and Ricchi scalar,

$$R_{\text{lin}}^{ab} = \frac{1}{2} (-\square \phi^{ab} + \partial^a \partial^c \phi^{cb} + \partial^b \partial^c \phi^{ca} - \partial^a \partial^b \phi^{cc}), \quad (4.25)$$

$$R_{\text{lin}} = \partial^a \partial^b \phi^{ab} - \square \phi^{aa}. \quad (4.26)$$

Using BRST transformations (4.14)-(4.19) and relations (4.21), we obtain

$$s\bar{c}^a = b^a, \quad s\bar{c} = b, \quad sb^a = 0, \quad sb = 0. \quad (4.27)$$

From (4.22),(4.27), we see that the fields b^a, b can really be considered as Nakanishi-Laudrup fields.

4.3 Arbitrary spin- s conformal field

BRST symmetries. For spin- s conformal field, a generating form of BRST invariant Lagrangian is given in (4.3). This Lagrangian is invariant under the following BRST and anti-BRST transformations:

$$s|\phi\rangle = G|c\rangle, \quad s|c\rangle = 0, \quad s|\bar{c}\rangle = \bar{L}|\phi\rangle, \quad (4.28)$$

$$\bar{s}|\phi\rangle = G|\bar{c}\rangle, \quad \bar{s}|c\rangle = -\bar{L}|\phi\rangle, \quad \bar{s}|\bar{c}\rangle = 0, \quad (4.29)$$

where the operators G, \bar{L} are given by

$$G \equiv \alpha \partial - e_1 - \alpha^2 \frac{1}{2N_\alpha + d - 2} \bar{e}_1, \quad (4.30)$$

$$\bar{L} \equiv \bar{\alpha} \partial - \frac{1}{2} \alpha \partial \bar{\alpha}^2 - \bar{e}_1 \Pi^{[1,2]} + \frac{1}{2} e_1 \bar{\alpha}^2, \quad (4.31)$$

$$e_1 = \alpha^z e_z \square, \quad \bar{e}_1 = -e_z \bar{\alpha}^z, \quad e_z = \left(\frac{2s + d - 4 - N_z}{2s + d - 4 - 2N_z} \right)^{1/2}, \quad (4.32)$$

while the operator $\Pi^{[1,2]}$ is defined in Appendix. It is easy to verify that BRST and anti-BRST transformations (4.28), (4.29) are off-shell nilpotent. In the case under consideration, the Nakanishi-Laudrup field $|b\rangle$ is defined by the relation $|b\rangle \equiv \bar{L}|\phi\rangle$. Using this relation and (4.28), (4.29), we find the relations $s\bar{c} = |b\rangle, s|b\rangle = 0, \bar{s}|c\rangle = -|b\rangle, \bar{s}|b\rangle = 0$, which demonstrate that the ket-vector $|b\rangle$ can really be considered as the Nakanishi-Laudrup field.

To illustrate a structure of Lagrangian (4.3), we use the decomposition of the ket-vectors into scalar, vector, and tensor fields of the $so(d-1, 1)$ algebra,

$$|\phi\rangle = \sum_{s'=0}^s \frac{\alpha_z^{s-s'}}{\sqrt{(s-s')!}} |\phi^{s'}\rangle, \quad |\phi^{s'}\rangle \equiv \frac{1}{s'!} \alpha^{a_1} \dots \alpha^{a_{s'}} \phi^{a_1 \dots a_{s'}} |0\rangle. \quad (4.33)$$

$$|c\rangle = \sum_{s'=0}^{s-1} \frac{\alpha_z^{s-1-s'}}{\sqrt{(s-1-s')!}} |c^{s'}\rangle, \quad |c^{s'}\rangle \equiv \frac{1}{s'!} \alpha^{a_1} \dots \alpha^{a_{s'}} c^{a_1 \dots a_{s'}} |0\rangle. \quad (4.34)$$

$$|\bar{c}\rangle = \sum_{s'=0}^{s-1} \frac{\alpha_z^{s-1-s'}}{\sqrt{(s-1-s')!}} |\bar{c}^{s'}\rangle, \quad |\bar{c}^{s'}\rangle \equiv \frac{1}{s'!} \alpha^{a_1} \dots \alpha^{a_{s'}} \bar{c}^{a_1 \dots a_{s'}} |0\rangle. \quad (4.35)$$

Note that tensorial gauge fields in (4.33) are double-traceless tensor fields of the $so(d-1, 1)$ algebra, while tensorial Faddeev-Popov fields in (4.34), (4.35) are traceless tensor fields of the $so(d-1, 1)$ algebra. Plugging ket-vectors (4.33)-(4.35) into (4.3) we get the following representation for Lagrangian (4.3):

$$\begin{aligned} \mathcal{L}_{\text{tot}} &= \sum_{s'=0}^s \mathcal{L}^{s'} + \sum_{s'=0}^{s-1} \mathcal{L}_{\text{FP}}^{s'}, \\ \mathcal{L}^{s'} &= \frac{1}{2s'!} \left(\phi^{a_1 \dots a_{s'}} \square^{\nu_{s'}} \phi^{a_1 \dots a_{s'}} - \frac{s'(s'-1)}{4} \phi^{aaa_3 \dots a_{s'}} \square^{\nu_{s'}} \phi^{bba_3 \dots a_{s'}} \right), \\ \mathcal{L}_{\text{FP}}^{s'} &= \frac{1}{s'!} \bar{c}^{a_1 \dots a_{s'}} \square^{\nu_{s'}+1} c^{a_1 \dots a_{s'}}, \quad \nu_{s'} = s' + \frac{d-4}{2}. \end{aligned} \quad (4.36)$$

Partition function. For the computation of a partition function, it is convenient to decompose the double-traceless ket-vector $|\phi\rangle$ into two traceless ket-vectors $|\phi_{\text{I}}\rangle, |\phi_{\text{II}}\rangle$,

$$\begin{aligned} |\phi\rangle &= |\phi_{\text{I}}\rangle + \alpha^2 \mathcal{N} |\phi_{\text{II}}\rangle, \quad \bar{\alpha}^2 |\phi_{\text{I}}\rangle = 0, \quad \bar{\alpha}^2 |\phi_{\text{II}}\rangle = 0, \\ \mathcal{N} &\equiv ((2s+d-4-2N_z)(2s+d-6-2N_z))^{-1/2}. \end{aligned} \quad (4.37)$$

Plugging (4.37) into (4.3), we get

$$\mathcal{L}_{\text{tot}} = \frac{1}{2} \langle \phi_{\text{I}} | \square^\nu | \phi_{\text{I}} \rangle - \frac{1}{2} \langle \phi_{\text{II}} | \square^\nu | \phi_{\text{II}} \rangle + \langle \bar{c} | \square^\nu | c \rangle. \quad (4.38)$$

The decomposition of the ket-vectors $|\phi_{\text{I,II}}\rangle$ into scalar, vector and tensor fields $\phi_{\text{I,II}}^{a_1 \dots a_{s'}}$ takes the form

$$\begin{aligned} |\phi_{\text{I}}\rangle &= \sum_{s'=0}^s \frac{\alpha_z^{s-s'}}{\sqrt{(s-s')!}} |\phi_{\text{I}}^{s'}\rangle, \quad |\phi_{\text{II}}\rangle = \sum_{s'=0}^{s-2} \frac{\alpha_z^{s-2-s'}}{\sqrt{(s-2-s')!}} |\phi_{\text{II}}^{s'}\rangle, \\ |\phi_{\text{I,II}}^{s'}\rangle &= \frac{1}{s'!} \alpha^{a_1} \dots \alpha^{a_{s'}} \phi_{\text{I,II}}^{a_1 \dots a_{s'}} |0\rangle. \end{aligned} \quad (4.39)$$

Note that the fields $\phi_{\text{I,II}}^{a_1 \dots a_{s'}}$ are totally symmetric traceless tensor fields of the $so(d-1, 1)$ algebra. In terms of the scalar, vector, and traceless tensor fields, Lagrangian (4.38) can be represented as

$$\mathcal{L}_{\text{tot}} = \sum_{s'=0}^s \mathcal{L}_{\text{I}}^{s'} - \sum_{s'=0}^{s-2} \mathcal{L}_{\text{II}}^{s'} + \sum_{s'=0}^{s-1} \mathcal{L}_{\text{FP}}^{s'}, \quad (4.40)$$

$$\mathcal{L}_I^{s'} = \frac{1}{2s'!} \phi_1^{a_1 \dots a_{s'}} \square^{\nu_{s'}} \phi_1^{a_1 \dots a_{s'}}, \quad \mathcal{L}_{II}^{s'} = \frac{1}{2s'!} \phi_{II}^{a_1 \dots a_{s'}} \square^{\nu_{s'}+2} \phi_{II}^{a_1 \dots a_{s'}}, \quad (4.41)$$

$$\mathcal{L}_{FP}^{s'} = \frac{1}{s'!} \bar{c}^{a_1 \dots a_{s'}} \square^{\nu_{s'}+1} c^{a_1 \dots a_{s'}}. \quad (4.42)$$

From (4.40)-(4.42), we see that the partition function is given by

$$Z = \prod_{s'=0}^{s-1} (D^{s'} D^{s'})^{\nu_{s'}+1} / \prod_{s'=0}^s (D^{s'})^{\nu_{s'}} \prod_{s'=0}^{s-2} (D^{s'})^{\nu_{s'}+2}, \quad (4.43)$$

$$D^{s'} \equiv (\det(-\square))^{1/2}, \quad (4.44)$$

where, in relation (4.44), determinant of the Laplace operator is evaluated on space of rank- s' traceless tensor field. It is easy to see that partition function (4.43) can be simplified as follows

$$Z = \frac{(D^{s-1})^{\nu_{s-1}+1}}{(D^s)^{\nu_s}}, \quad \nu_s = s + \frac{d-4}{2}. \quad (4.45)$$

Partition function (4.45) was obtained by different method in earlier literature in Refs.[37, 14]). Also, in Ref.[14] it was noted that it is helpful to use the following relation

$$D^{s'} = D^{s'\perp} D^{s'-1}, \quad (4.46)$$

where $D^{s'\perp}$ takes the same form as in (4.44), while the determinant of Laplace operator is evaluated on space of traceless and divergence-free rank- s' tensor field. Namely, by using (4.46), partition function (4.45) can be cast into the form

$$Z = \frac{1}{(D^{s\perp})^{(d-4)/2}} \prod_{s'=0}^{s-1} \frac{D^{s'\perp}}{D^{s\perp}}. \quad (4.47)$$

In Ref.[14], expression (4.47) was generalized to a partition function for a conformal field in (A)dS space. Computation of the partition function for the conformal field in (A)dS by various methods may be found in Refs.[14, 38].

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Appendix A Notation and conventions

The vector indices of the $so(d, 1)$ algebra take the values $A, B, C, E = 0, 1, \dots, d$, while the vector indices of the $so(d-1, 1)$ algebra take the values $a, b, c, e = 0, 1, \dots, d-1$. To simplify our expressions we drop the flat metrics $\eta^{AB} = (-, +, \dots, +)$ and $\eta^{ab} = (-, +, \dots, +)$ in scalar product, i.e., we use $X^A Y^A \equiv \eta_{AB} X^A Y^B$, $X^a Y^a \equiv \eta_{ab} X^a Y^b$.

Covariant derivative with flat indices D^A is defined by the relations $D^A = \eta^{AB} D_B$,

$$D_A \equiv e_A^{\underline{m}} D_{\underline{m}}, \quad D_{\underline{m}} \equiv \partial_{\underline{m}} + \frac{1}{2} \omega_{\underline{m}}^{AB} M^{AB}, \quad M^{AB} = \alpha^A \bar{\alpha}^B - \alpha^B \bar{\alpha}^A, \quad (A.1)$$

$\partial_{\underline{m}} = \partial / \partial x^{\underline{m}}$, $\underline{m} = 0, 1, \dots, d$, where $x^{\underline{m}}$ are base manifold coordinates of (A)dS space, $e_A^{\underline{m}}$ is inverse of vielbein $e_{\underline{m}A}$, $e_{\underline{m}A}^{\underline{m}} e_{\underline{m}B}^{\underline{m}} = \eta_{AB}$, $D_{\underline{m}}$ covariant derivative with Lorentz connection $\omega_{\underline{m}}^{AB}$,

and M^{AB} is spin operator of Lorentz algebra $so(d, 1)$. Contravariant tensor field in (A)dS with flat indices, $\Phi^{A_1 \dots A_s}$, is related to contravariant tensor field with base manifold indices, $\Phi^{\underline{m}_1 \dots \underline{m}_s}$, in a standard way: $\Phi^{A_1 \dots A_s} \equiv e^{\underline{m}_1}_{A_1} \dots e^{\underline{m}_s}_{A_s} \Phi^{\underline{m}_1 \dots \underline{m}_s}$. D'Alembert operator in (A)dS is defined by the relation

$$\square_{(A)dS} \equiv D^A D_A + \omega^{AB} D_B, \quad \omega^{ABC} \equiv e^{\underline{A}}_{\underline{m}} \omega^{\underline{B}\underline{C}}_{\underline{m}}, \quad e \equiv \det e^{\underline{A}}_{\underline{m}}. \quad (\text{A.2})$$

Derivative $D_{\underline{m}}$ (A.1) is defined in space of ket-vectors constricted out of α^A . Derivative acting on tensor field are denoted by $\mathcal{D}_{\underline{m}}$. Actions of such derivative on vector field with flat indices is defined in a standard way

$$\mathcal{D}_{\underline{m}} \phi^A = \partial_{\underline{m}} \phi^A + \omega^{\underline{A}\underline{B}}_{\underline{m}}(e) \phi^{\underline{B}}. \quad (\text{A.3})$$

In place of $\mathcal{D}_{\underline{m}}$, we prefer to use a derivative with flat indices, \mathcal{D}^A ,

$$\mathcal{D}_A \equiv e^{\underline{m}}_A \mathcal{D}_{\underline{m}}, \quad \mathcal{D}^A = \eta^{AB} \mathcal{D}_B, \quad [\mathcal{D}^A, \mathcal{D}^B] \phi^C = R^{ABCE} \phi^E, \quad (\text{A.4})$$

$\mathcal{D}^2 \equiv \mathcal{D}^A \mathcal{D}_A$, where Riemann tensor of (A)dS space is given by

$$R^{ABCE} = \rho(\eta^{AC} \eta^{BE} - \eta^{AE} \eta^{BC}), \quad \rho = \frac{\epsilon}{R^2}, \quad \epsilon = \begin{cases} 1 & \text{for dS} \\ -1 & \text{for AdS} \end{cases} \quad (\text{A.5})$$

Creation operators α^A , ζ and the respective annihilation operators $\bar{\alpha}^A$, $\bar{\zeta}$ are referred to as oscillators in this paper. Commutation relations, the vacuum $|0\rangle$, and hermitian conjugation rules are fixed by the relations

$$[\bar{\alpha}^A, \alpha^B] = \eta^{AB}, \quad [\bar{\zeta}, \zeta] = 1, \quad \bar{\alpha}^A |0\rangle = 0, \quad \bar{\zeta} |0\rangle = 0, \quad \alpha^{A\dagger} = \bar{\alpha}^A, \quad \zeta^\dagger = \bar{\zeta}. \quad (\text{A.6})$$

Oscillators α^A , $\bar{\alpha}^A$ and ζ , $\bar{\zeta}$, transform in the respective vector and scalar representation of the $so(d, 1)$ algebra. Derivatives with respect to space-time coordinates x^a , z are denoted by $\partial^a \equiv \eta^{ab} \partial / \partial x^b$, $\partial_z \equiv \partial / \partial z$. In basis of the $so(d-1, 1)$ algebra, we use the decompositions $\alpha^A = \alpha^a, \alpha^z$ and $\eta^{AB} = \eta^{ab}, \eta^{zz}$, where $\eta^{zz} = 1$. We adopt the following notation for the scalar product of oscillators and derivatives

$$\alpha \mathbf{D} \equiv \alpha^A D_A, \quad \bar{\alpha} \mathbf{D} \equiv \bar{\alpha}^A D_A, \quad \alpha^2 \equiv \alpha^A \alpha_A, \quad \bar{\alpha}^2 \equiv \bar{\alpha}^A \bar{\alpha}_A, \quad (\text{A.7})$$

$$\alpha \partial \equiv \alpha^a \partial_a, \quad \bar{\alpha} \partial \equiv \bar{\alpha}^a \partial_a, \quad \alpha^2 \equiv \alpha^a \alpha_a, \quad \bar{\alpha}^2 \equiv \bar{\alpha}^a \bar{\alpha}_a, \quad (\text{A.8})$$

$$N_\alpha \equiv \alpha^A \bar{\alpha}_A, \quad N_\alpha \equiv \alpha^a \bar{\alpha}_a, \quad N_\zeta \equiv \zeta \bar{\zeta}, \quad N_z \equiv \alpha^z \bar{\alpha}_z, \quad (\text{A.9})$$

$$\Pi^{[1,2]} \equiv 1 - \alpha^2 \frac{1}{2(2N_\alpha + d)} \bar{\alpha}^2, \quad \bar{\Pi}^{[1,2]} \equiv 1 - \bar{\alpha}^2 \frac{1}{2(2N_\alpha + d + 1)} \alpha^2, \quad (\text{A.10})$$

$$\square \equiv \partial^a \partial_a, \quad \mu \equiv 1 - \frac{1}{4} \alpha^2 \bar{\alpha}^2, \quad \bar{\mu} \equiv 1 - \frac{1}{4} \bar{\alpha}^2 \alpha^2. \quad (\text{A.11})$$

Bra-vectors and ket-vectors of Faddeev-Popov fields satisfy the relations $\langle c| = |c\rangle^\dagger$, $\langle \bar{c}| = -|\bar{c}\rangle^\dagger$, while Faddeev-Popov scalar, vector, and tensor fields satisfy the following hermitian conjugation conditions: $c^{A_1 \dots A_{s'}\dagger} = c^{A_1 \dots A_{s'}}$, $\bar{c}^{A_1 \dots A_{s'}\dagger} = -\bar{c}^{A_1 \dots A_{s'}}$. To illustrate these hermitian conjugation rules we consider Faddeev-Popov vector fields and note that, if ket-vectors are given by $|c\rangle = \alpha^A c^A |0\rangle$, $|\bar{c}\rangle = \alpha^A \bar{c}^A |0\rangle$, then the respective bra-vectors are given by $\langle c| = \langle 0| \bar{\alpha}^A c^A$, $\langle \bar{c}| = \langle 0| \bar{\alpha}^A \bar{c}^A$.

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